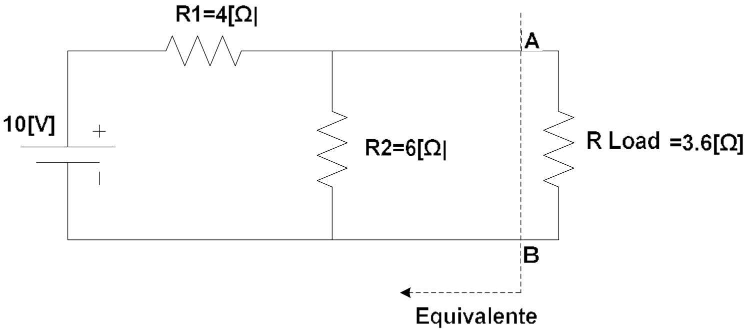
ejercicios resueltos con Teoremas de Thevenin y Norton

circuitos electricos I (Universidad Mayor Real y Pontificia San Francisco Xavier de Chuquisaca)

THEVENIN Y NORTON

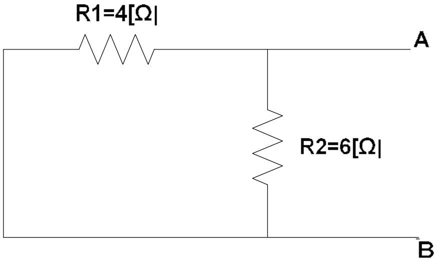
1. Problema

Encuentre el equivalente de Thevenin del circuito en los terminales a y b, y encuentre la corriente IL como el voltaje VL



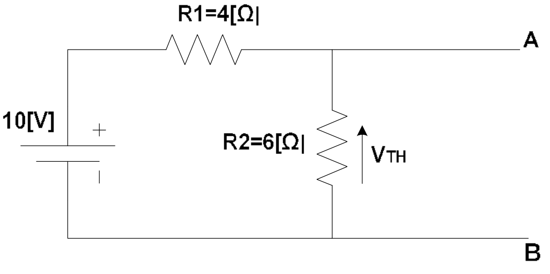
Solución:

Quitar la carga RL y anular la fuente de energía

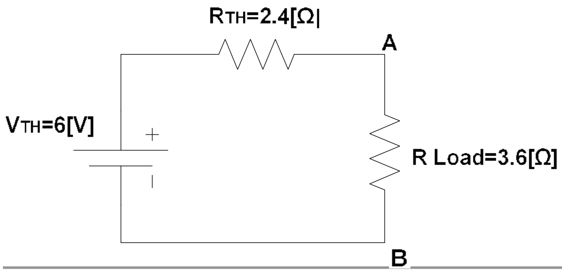


$$R_{TH} = R1 // R2 = \frac{R1R2}{R1+R2} = \frac{4 \cdot 6}{4+6} = \frac{24}{10} = 2.4\Omega$$

Calcular voltaje Thevenin



$$V_{TH} = 10\frac{6}{6+4} = 6V$$

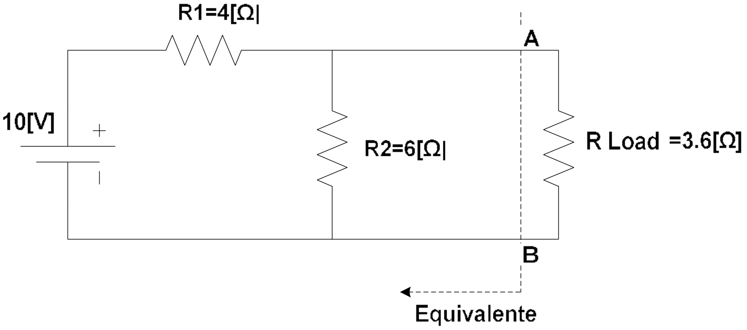


$$i_L = \frac{V_{TH}}{R_{TH}+R_L} = \frac{6}{2.4+3.6} = \frac{6}{6} = 1[A]$$

$$V_L = i_L \cdot R_L = 1 \cdot 3.6 = 3.6[V]$$

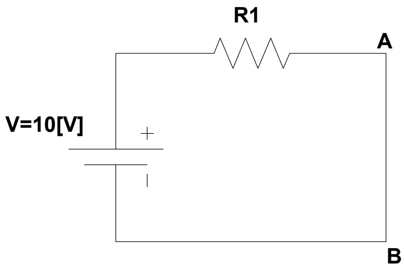
2. Problema

Calcule la corriente i_L por medio del teorema de Norton.



Solución:

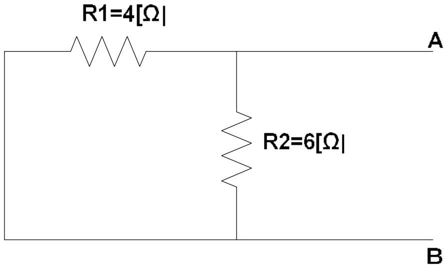
Encuentre I_N. Haga un corto entre las terminales A-B, que cortocircuite R1 y R2 en paralelo.



La única resistencia del circuito es R1 en serie con la fuente V

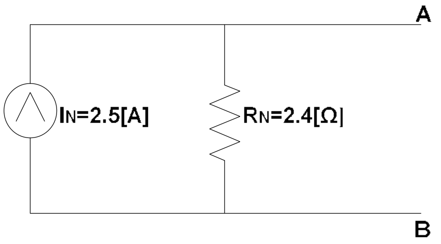
$$I_N = \frac{V}{R_1} = \frac{10}{4} = 2.5[A]$$

Encuentre R_N. Abra los terminales A-B y póngase V en cortocircuito. R1 y R2 están en paralelo, así que:

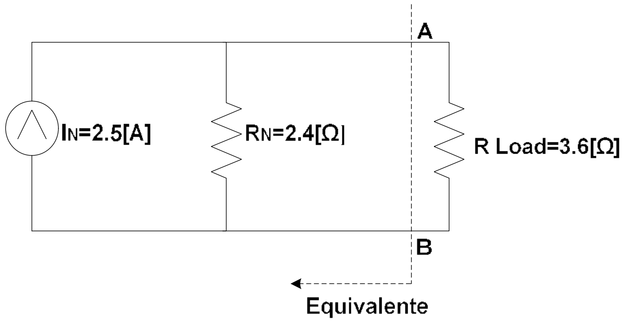


$$R_N = \frac{4 \cdot 6}{4+6} = \frac{24}{10} = 2.4[\Omega]$$

Note que R_N es igual a R_{TH}. La flecha en la fuente de corriente muestra la dirección convencional de la corriente de la terminal A hacia la terminal B, como el circuito original.



Encuentre Ic. Reconecte Rl a los terminales A-B. La fuente aún produce 2.5 [A], pero la corriente entre las dos ramas R_N y Rl.

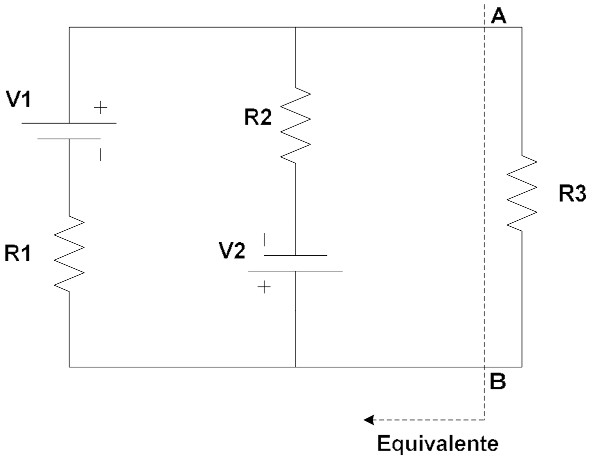


$$I_L = \frac{R_N}{R_N + R_L} I_N = \frac{2.4}{2.4+3.6} \cdot 2.5 = \frac{2.4}{6} \cdot 2.5 = 1[A]$$

$V_L = I_L * R_L = 1[A] * 3.6\ [\Omega] = 3.6[V]$

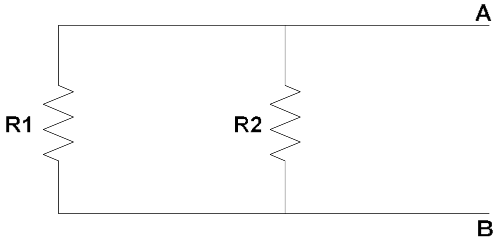
3. Problema

Hallar la corriente que circula por la resistencia R3, empleando el teorema de Thevenin.



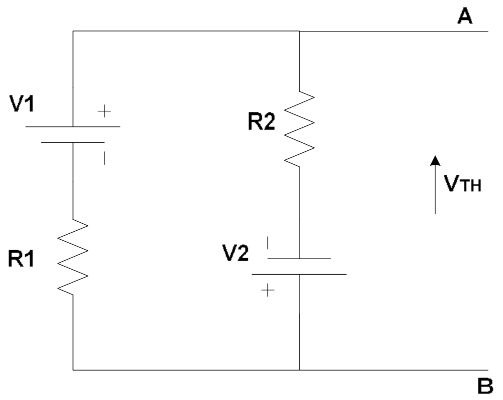
Solución:

1º anular las fuentes de energía y quitar la carga R3



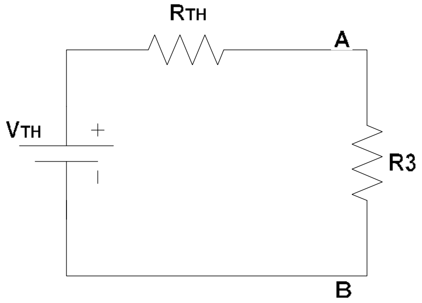
$$R_{TH} = R1 // R2 = \frac{R1 \cdot R2}{R1 + R2}$$

2º buscar voltaje Thevenin



$$V_{TH} = V1 \frac{R2}{R1 + R2} - V2 \frac{R1}{R1 + R2} = \frac{V1 \cdot R2 - V2 \cdot R1}{R1 \cdot R2}$$

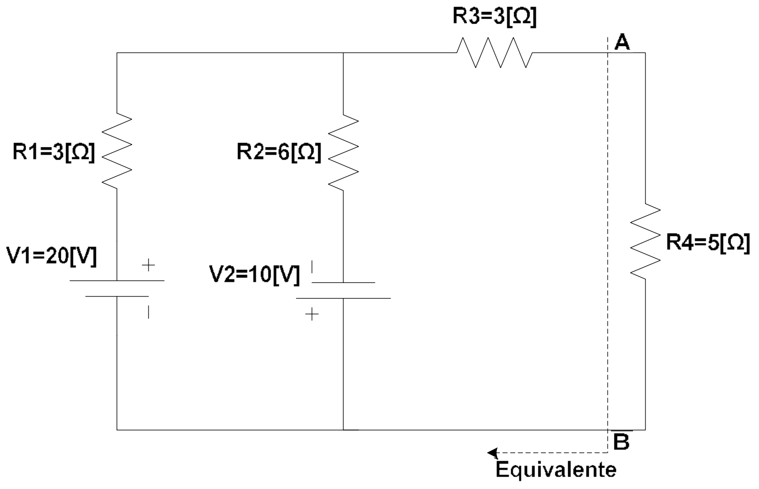
3º Cálculo de la intensidad que circula por R3



$$i_3 = \frac{\dfrac{V1 \cdot R2 - V2R1}{R1 \cdot R2}}{R_{TH} + R3} = \frac{\dfrac{V1 \cdot R2 - V2 \cdot R1}{R1 \cdot R2}}{\dfrac{R1 \cdot R2}{R1 + R2} + R3} = \frac{V1 \cdot R2 - V2 \cdot R1}{R1 \cdot R2 + R3 \cdot R1 + R2 \cdot R3}$$

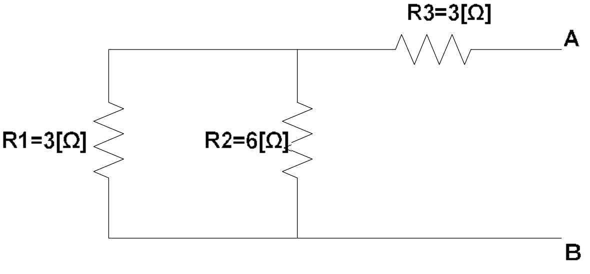
4. Problema:

Calcula la intensidad I que pasará por la resistencia 5Ω utilizando Thevenin y Norton.



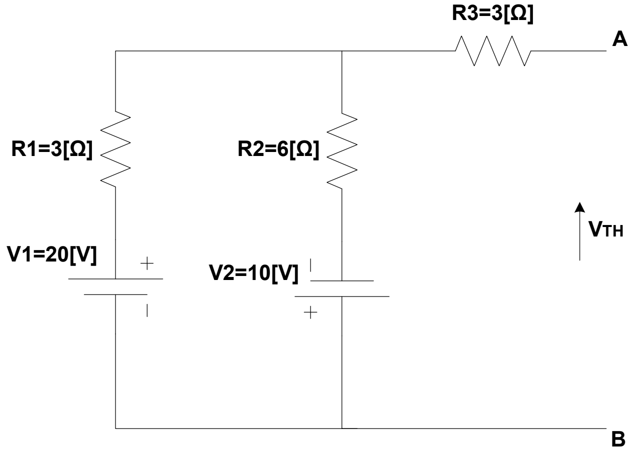
Solución:

1º anular las fuentes de energía y quitar la carga de 5Ω



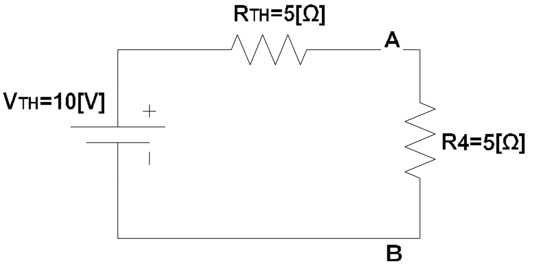
$$R_{TH} = 3 // 6 + 3 = \frac{3 \cdot 6}{3+6} + 3 = \frac{18}{9} + 3 = 2 + 3 = 5[\Omega]$$

2º buscar voltaje Thevenin



$$V_{TH} = 20 \cdot \frac{6}{3+6} - 10 \cdot \frac{3}{3+6} = \frac{120}{9} - \frac{30}{9} = 10[V]$$

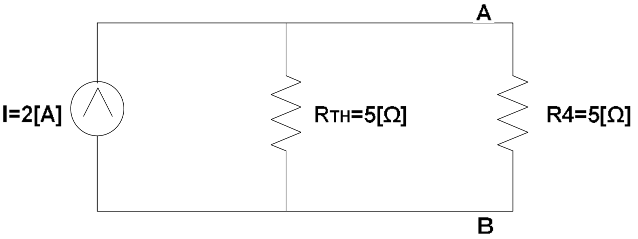
3º cálculo de la intensidad que circula por R=5Ω



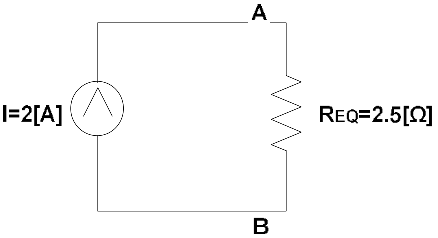
$$I_L = \frac{10}{5+5} = \frac{10}{10} = 1[A]$$

4° teorema Norton

$$I_N = \frac{V_{TH}}{R_{TH}} = \frac{10}{5} = 2[A]$$ y RTH=RN=5[Ω]



$$R_{EQ} = \frac{5 \cdot 5}{5+5} = \frac{25}{10} = 2.5[\Omega]$$

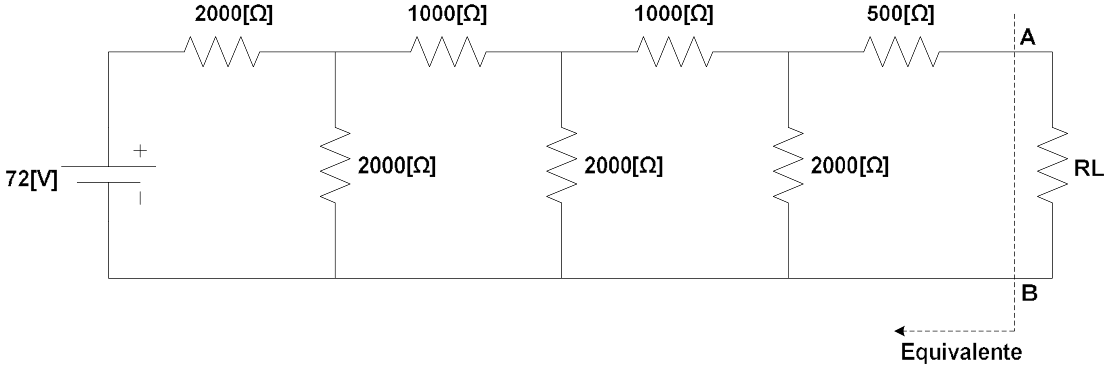


$$V = 2 \cdot 2.5 = 5[V]$$

$$I_L = \frac{5[V]}{5[\Omega]} = 1[A]$$

5. Problema

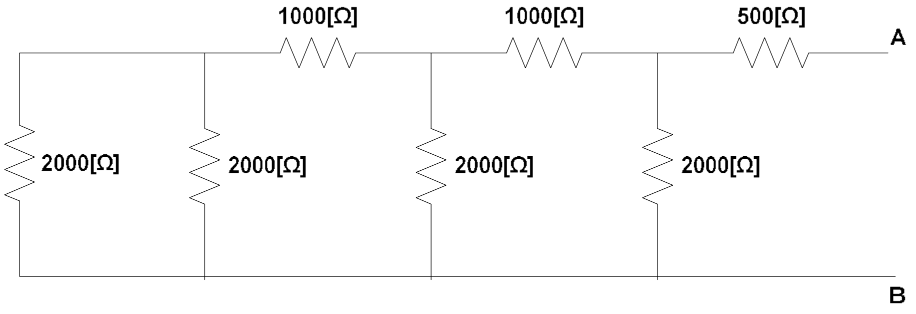
Teorema de Thevenin



Calcular IL cuando RL=1.5 [KΩ]
RL=3 [KΩ]
RL=4.5 [KΩ]

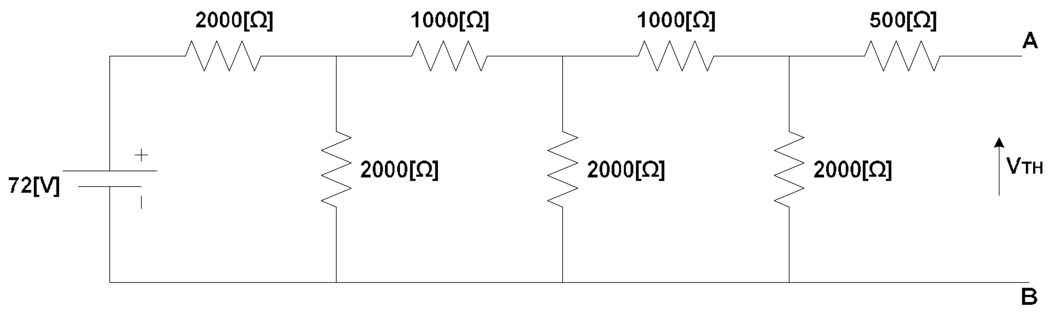
Solución:

1º anular las fuentes de energía y quitar la carga RL



R_{TH} = ((2KΩ//2KΩ + 1KΩ)// 2KΩ + 1KΩ)// 2KΩ + 0.5KΩ
R_{TH} = ((1KΩ + 1KΩ)// 2KΩ + 1KΩ)// 2KΩ + 0.5KΩ
R_{TH} = (2KΩ// 2KΩ + 1KΩ)// 2KΩ + 0.5KΩ
R_{TH} = (1KΩ + 1KΩ)// 2KΩ + 0.5KΩ
R_{TH} = 2KΩ// 2KΩ + 0.5KΩ
R_{TH} = 1KΩ + 0.5KΩ
R_{TH} = 1.5KΩ

2º buscar voltaje Thevenin



$$V_{TH} = Va \cdot \frac{2}{2+1} = Va \cdot \frac{2}{3}$$

$$Va = Vb \cdot \frac{2//3}{2//3+1} = Vb \cdot \frac{\frac{6}{5}}{\frac{6}{5}+1} = Vb \cdot \frac{\frac{6}{5}}{\frac{11}{5}} = Vb \cdot \frac{6}{11}$$

$$Vb = 72 \cdot \frac{((2+1)//2+1)//2}{((2+1)//2+1//2+2} = 72 \cdot \frac{(3//2+1)//2}{(3//2+1)//2+2} = 72 \cdot \frac{(\frac{6}{5}+1)//2}{(\frac{6}{5}+1)//2+2} =$$

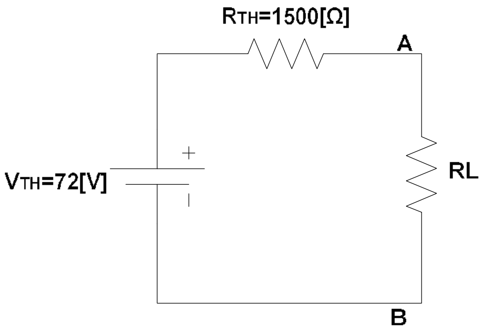
$$72 \cdot \frac{\frac{11}{5}//2}{\frac{11}{5}//2+2} = 72 \cdot \frac{\frac{22}{21}}{\frac{22}{21}+2} = 72 \cdot \frac{\frac{22}{21}}{\frac{64}{21}} = 72 \cdot \frac{22}{64} = \frac{99}{4}$$

Entonces:

$$Va = Vb \cdot \frac{6}{11} = \frac{99}{4} \cdot \frac{6}{11} = \frac{27}{2}$$

Y $V_{TH} = Va \cdot \frac{2}{3} = \frac{27}{2} \cdot \frac{2}{3} = 9[V]$ $\Rightarrow$ V_{TH}=9[V]

3º unir la carga al circuito equivalente conseguido



a) R_L=1.5[KΩ]

$$I_L = \frac{9}{1.5+1.5} = 3[mA]$$

b) R_L=3[KΩ]

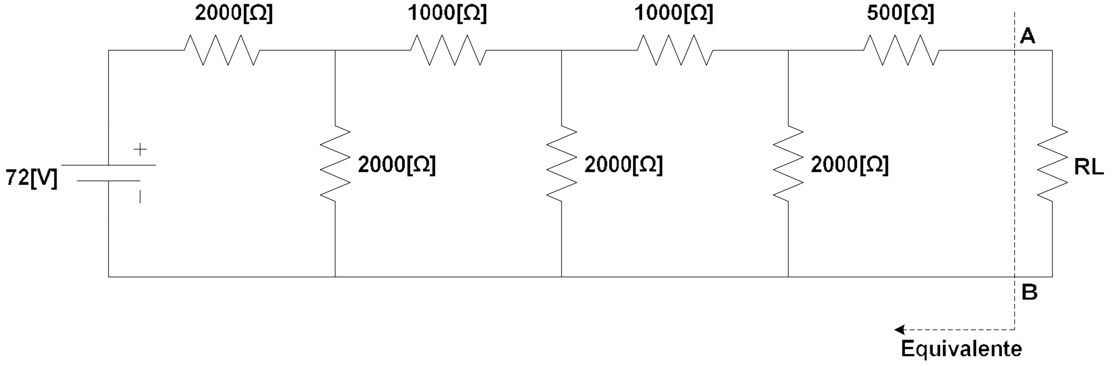
$$I_L = \frac{9}{1.5+3} = 2[mA]$$

c) R_L=4.5[KΩ]

$$I_L = \frac{9}{1.5+4.5} = 1.5[mA]$$

6. Problema

Teorema de Norton



Calcular IL cuando RL=1.5 [KΩ]
RL=3 [KΩ]
RL=4.5 [KΩ]

Solución:

1º y 2º Todo el proceso es el mismo que el anterior, así que volveremos anotar los datos finales:

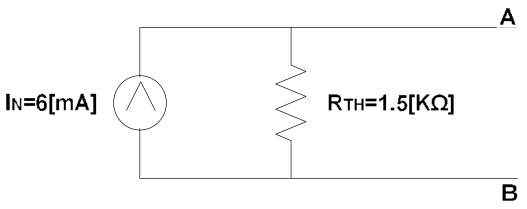
$$R_{TH} = 1.5[K\Omega]$$

$$V_{TH} = 9[V]$$

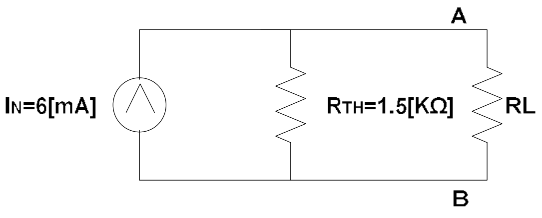
3° Encontrar la corriente Norton

$$I_N = \frac{V_{TH}}{R_{TH}} = \frac{9}{1.5K\Omega} = 6[mA]$$

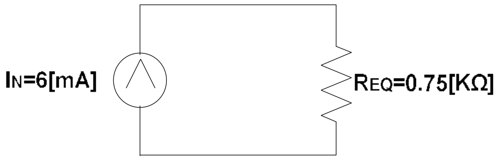
4° Unir la carga al circuito equivalente conseguido



a) RL=1.5[KΩ]



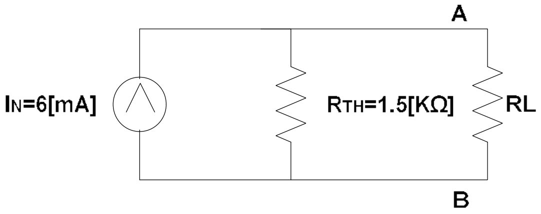
$$R_{EQ} = \frac{1.5 \cdot 1.5}{1.5 + 1.5} = \frac{2.25}{3} = 0.75[K\Omega]$$



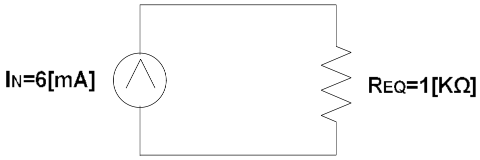
$$V = 6 \cdot 0.75 = 4.5[V]$$

$$I_L = \frac{4.5}{1.5} = 3[mA]$$

b) RL=3[KΩ]



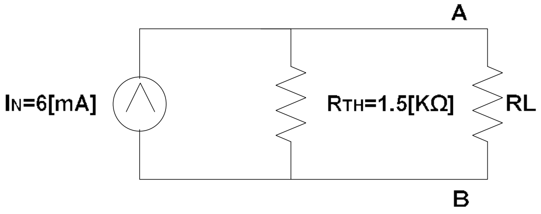
$$R_{EQ} = \frac{1.5 \cdot 3}{1.5 + 3} = \frac{4.5}{4.5} = 1[K\Omega]$$



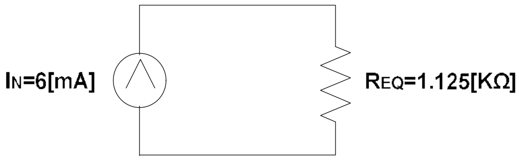
$$V = 6 \cdot 1 = 6[V]$$

$$I_L = \frac{6}{3} = 2[mA]$$

c) RL=4.5[KΩ]



$$R_{EQ} = \frac{1.5 \cdot 4.5}{1.5 + 4.5} = \frac{6.75}{6} = 1.125[K\Omega]$$

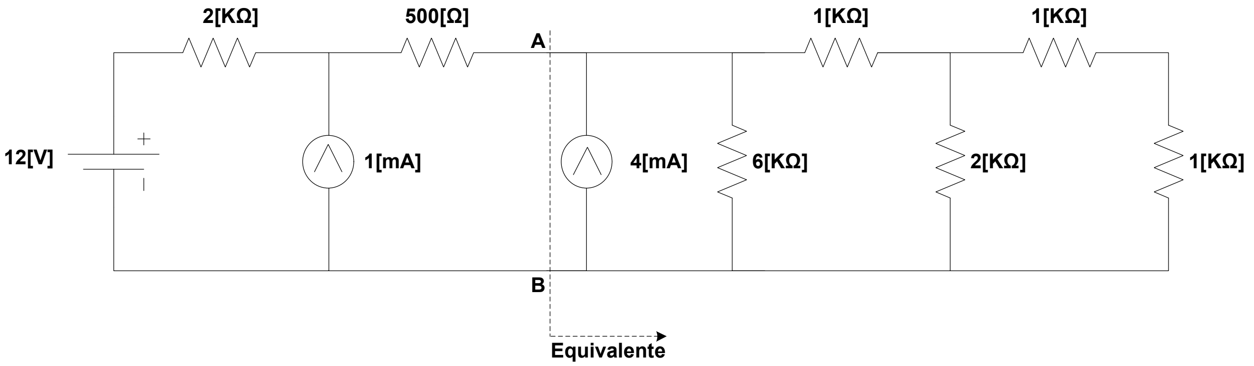


$$V = 6 \cdot 1.125 = 6.75[V]$$

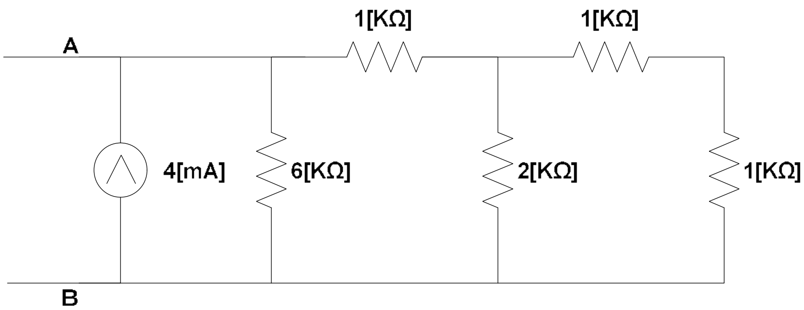
$$I_L = \frac{6.75}{4.5} = 1.5[mA]$$

7. Problema

Encontrar el equivalente de Thevenin del circuito a la derecha de la línea

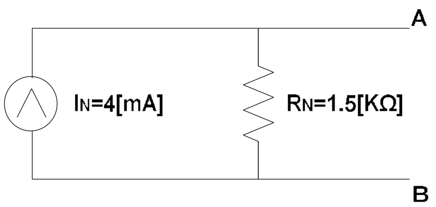


Solución:

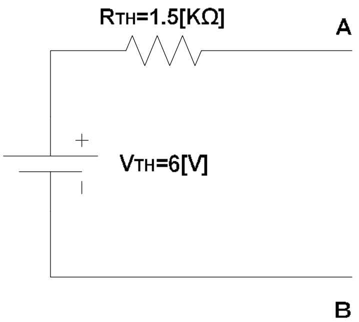


$R_{TH} = ((1K\Omega+1\ K\Omega)//2\ K\Omega+1\ K\Omega)//6\ K\Omega$
$R_{TH} = (2\ K\Omega//2\ K\Omega+1\ K\Omega)//6\ K\Omega$
$R_{TH} = (1K\Omega+1\ K\Omega)//6\ K\Omega$
$R_{TH} = 2K\Omega//6\ K\Omega$
$R_{TH} = 1.5K\Omega$

Es una disposición Norton



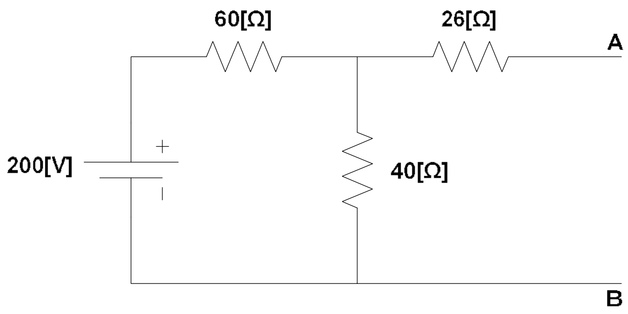
Entonces el equivalente de Thevenin será:



$R_{TH} = R_N = 1.5[K\Omega]$
$V_{TH} = I_N \times R_{TH} = 4[mA] \times 1.5[K\Omega] = 6[V]$

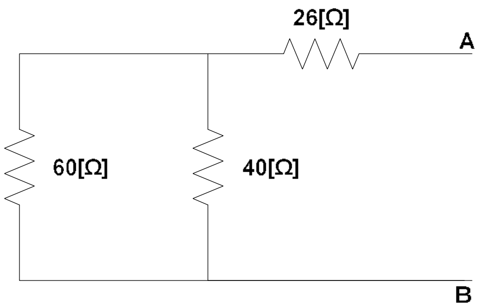
8. Problema

Colocar el equivalente Thevenin y de Norton del siguiente circuito



Solución:

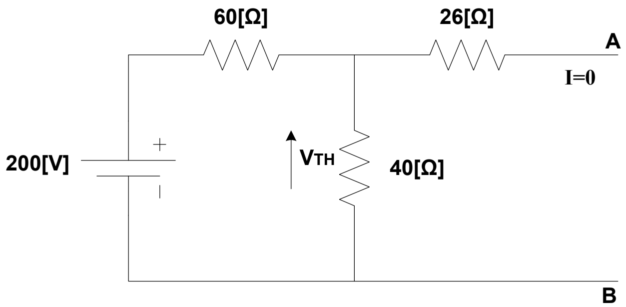
a. Calculamos la resistencia R_{TH} y R_N ($R_{TH}=R_N$)
Para ello, cortocircuitamos todas las fuentes:



$$R_{TH} = 60 // 40 + 26 = \frac{60 \cdot 40}{60+40} + 26$$

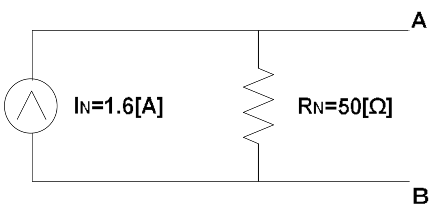
$$= \frac{2400}{100} + 26 = 50[\Omega]$$

b. Voltaje Thevenin, calculamos en circuito alto y eso implica resolver y ver que voltaje hay en R=40Ω



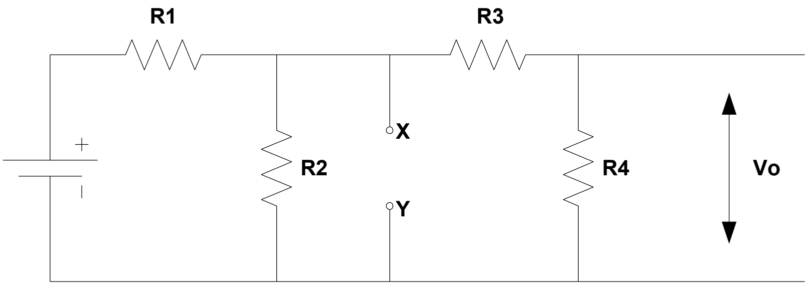
$$V_{TH} = 200 \cdot \frac{40}{40+60} = 200 \cdot \frac{40}{100} = 80[V]$$

c. I_N es $\frac{V_{TH}}{R_{TH}} = \frac{80[V]}{50[\Omega]} = 1.6[A]$



9. Problema

Calcule Vo por el teorema de Thevenin entre los puntos *x-y*



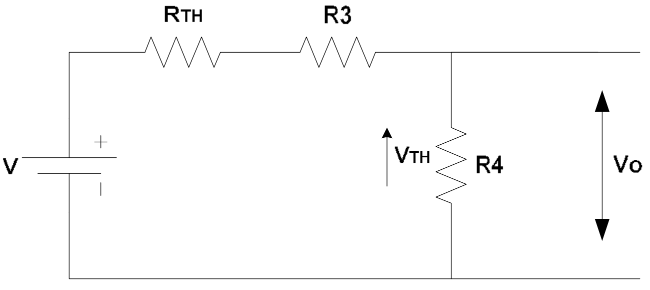
Solución:

1° Desconectamos lo que tenemos a ala derecha de los puntos x e y (es decir, estamos suponiendo que las resistencias R3 y R4, las hemos desconectado físicamente del circuito original)

$$V_{TH} = V\frac{R2}{R2+R1}$$

2° Resistencia equivalente Thevenin

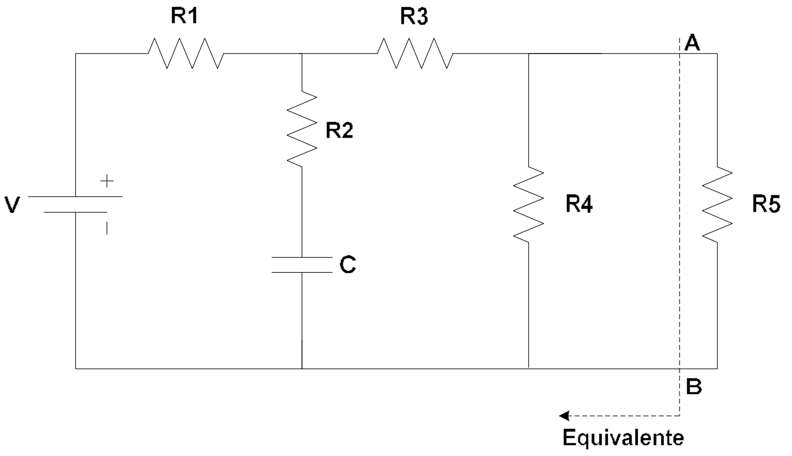
$$R_{TH} = R1 // R2 = \frac{R1R2}{R1+R2}$$



$$V_O = V_{TH}\frac{R4}{R_{TH}+R3+R4} = V\frac{R2}{R1+R2}\cdot\frac{R4}{\frac{R1\cdot R2}{R1+R2}+R3+R4}$$

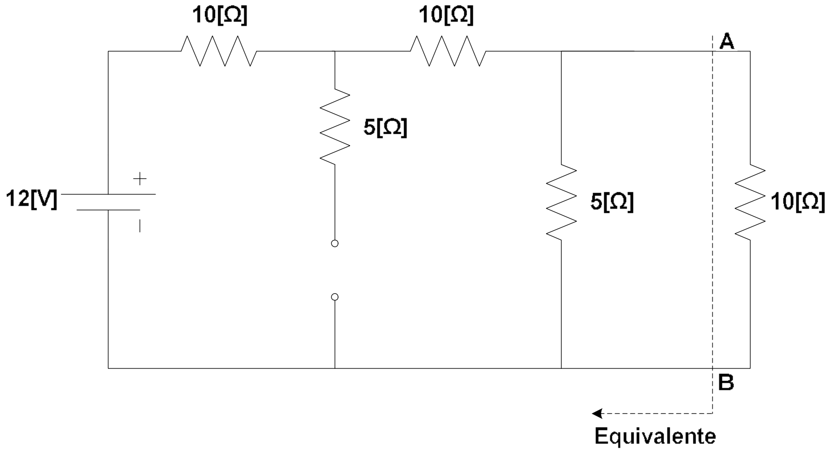
10. Problema

Determine el equivalente de Thevenin del circuito a la izquierda de los terminales A y B.



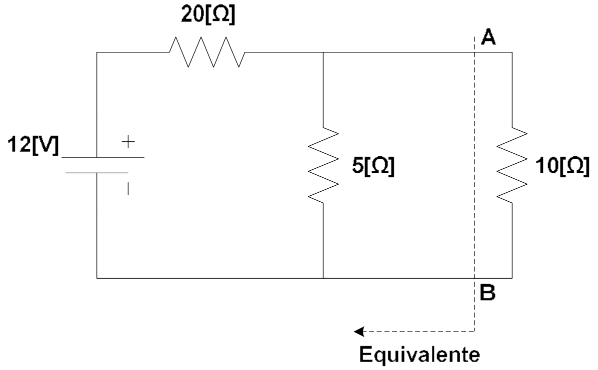
Solución:

Estado estacionario

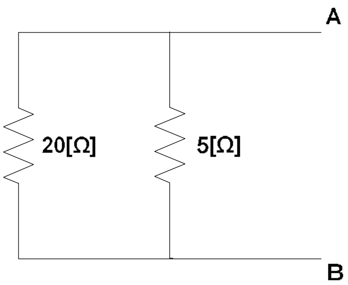


$$R_{EQ} = 10 + 10 = 20[\Omega]$$

Entonces, el circuito resulta:

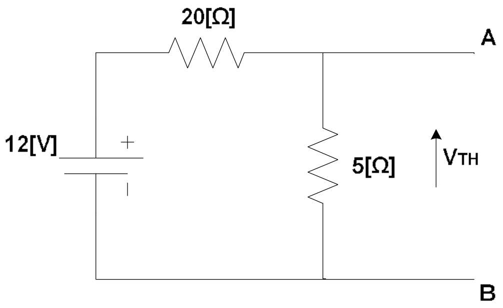


Quitar la carga de 10[Ω] y anular la fuente de energía



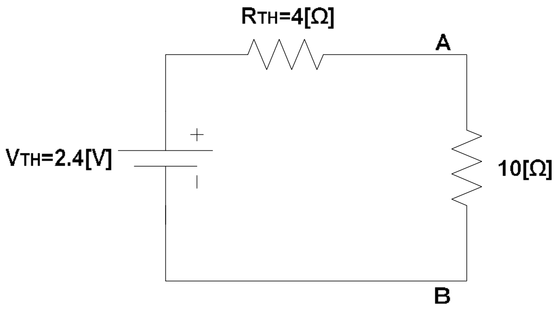
$$R_{TH} = 20 // 5 = \frac{20 \cdot 5}{20+5} = \frac{100}{25} = 4[\Omega]$$

Calcular el voltaje Thevenin:



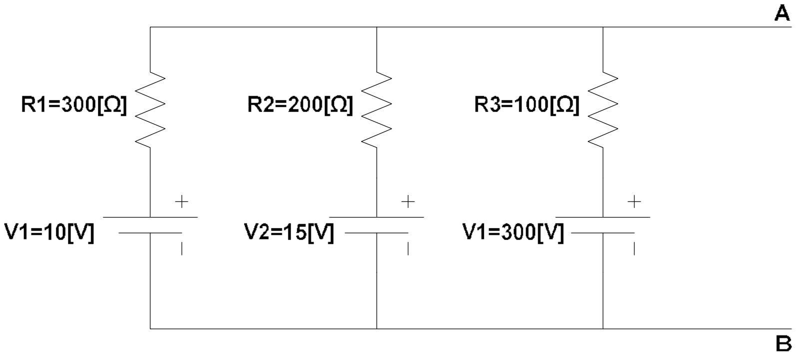
$$V_{TH} = 12 \cdot \frac{5}{20+5} = \frac{60}{25} = 2.4[V]$$

Entonces el equivalente Thevenin es:



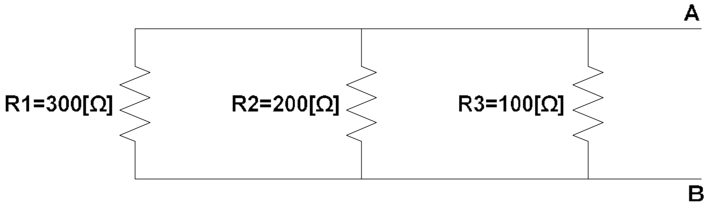
11. Problema

Determinar los equivalentes Thevenin y Norton de la red de los terminales A-B



Solución:

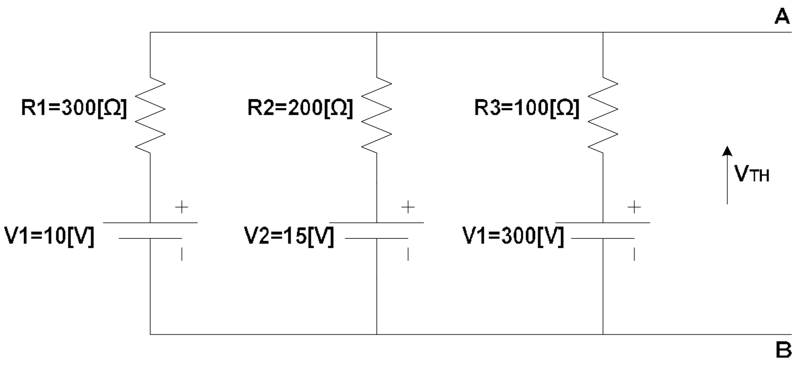
1° anular las fuentes de energía



$$R_{TH} = R1 // R2 // R3 = 300 // 200 // 100 = \frac{300 \cdot 200}{300 + 200} // 100 = \frac{60000}{500} // 100 = 120 // 100 =$$

$$= \frac{120 \cdot 100}{120 + 100} = \frac{12000}{220} = \frac{600}{11} = 54.54[\Omega]$$

2° buscar voltaje Thevenin



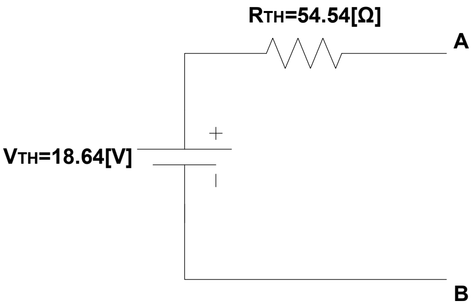
$$V_{TH} = -V1\frac{R2//R3}{R2//R3+R1} + V2\frac{R1//R3}{R1//R3+R2} + V3\frac{R1//R2}{R1//R2+R3} =$$

$$= -10\frac{200//100}{200//100+300} + 15\frac{300//100}{300//100+200} + 30\frac{300//200}{300//200+100} =$$

$$= -10\frac{200/3}{200/3+300} + 15\frac{75}{75+200} + 30\frac{120}{120+100} =$$

$$= -\frac{2000/3}{1100/3} + \frac{1125}{275} + \frac{3600}{220} = -\frac{20}{11} + \frac{45}{11} + \frac{180}{11} = \frac{205}{11} = 18.64[V]$$

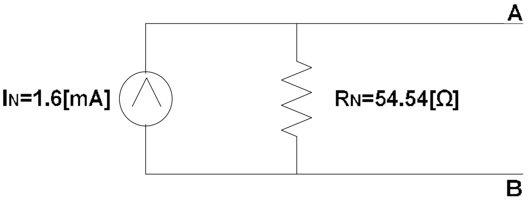
Equivalente Thevenin



Equivalente Norton

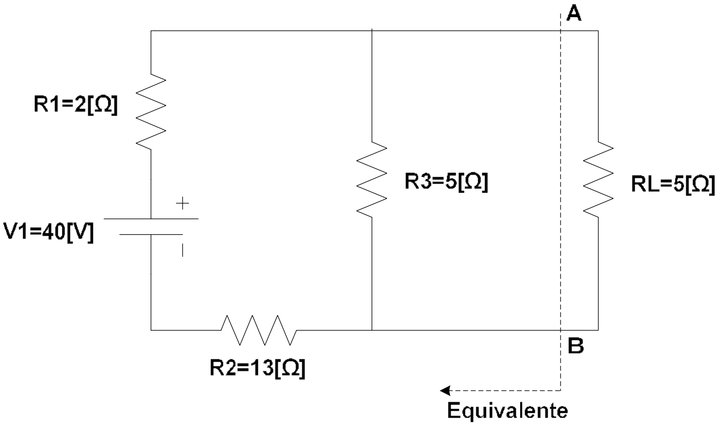
$$I_N = \frac{V_{TH}}{R_{TH}} = \frac{18.64}{54.54} = 0.34[A]$$

$$R_{TH} = R_N = 54.54[\Omega]$$



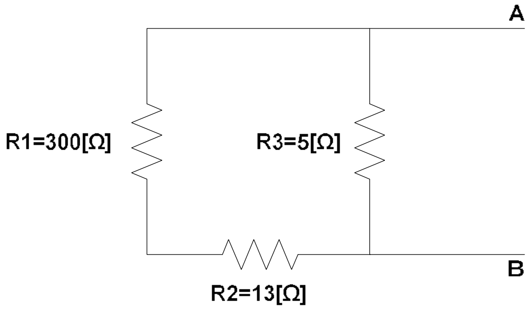
12. Problema

Calcular la corriente IL cuando RL=5[Ω] y RL=30[Ω]



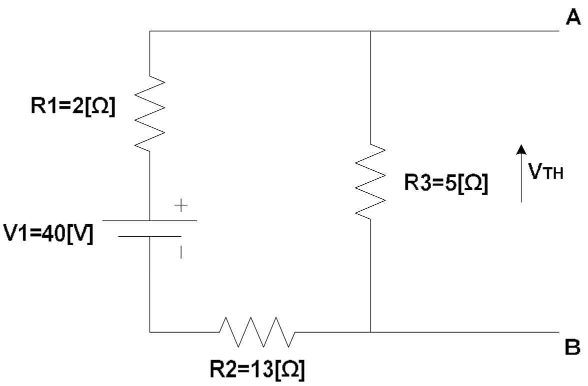
Solución:

Para determinar R_{TH}, se reducen todas las fuentes a cero reemplazándolas con un cortocircuito equivalente



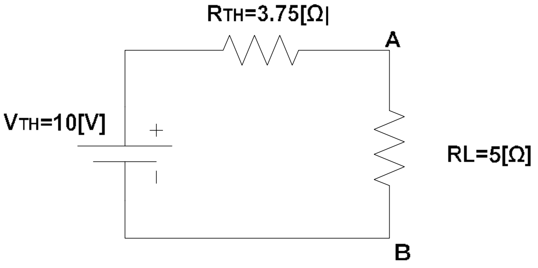
$$R_{TH} = R3//(R1+R2) = \frac{R3(R1+R2)}{R3+(R1+R2)} = \frac{5(13+2)}{5+(13+2)} = \frac{75}{20} = 3.75[\Omega]$$

Para determinar V_{TH}, se recolocan la fuentes y se determinan los voltajes de circuito abierto



$$V_{TH} = V \cdot R3 = 40 \cdot \frac{5}{(2+13)+5} = \frac{200}{20} = 10[V]$$

Reconectar RL a los terminales A-B

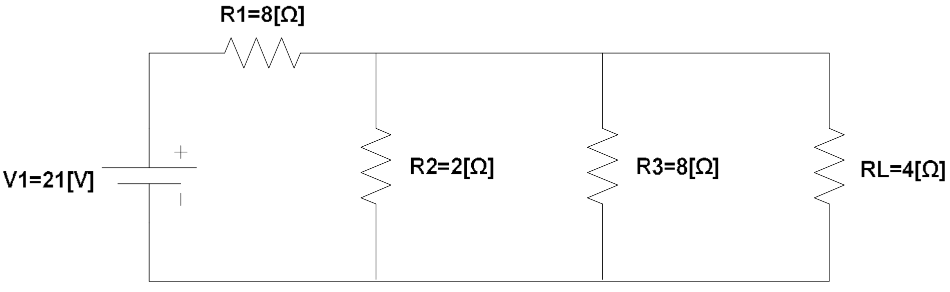


La corriente a través de RL es

$$I_L = \frac{V_{TH}}{R_{TH} + R_L} = \frac{10}{3.75+5} = 1.143[A]$$

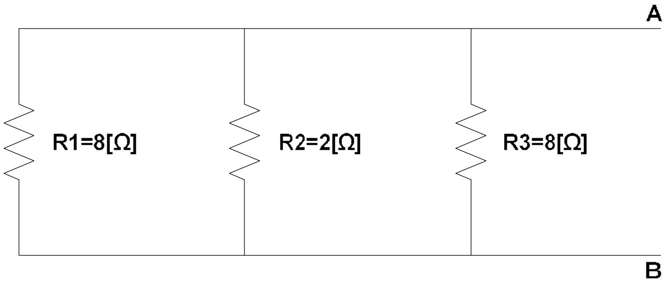
13. Problema

Encuentre la red de Thevenin para la porción de la red a la izquierda del resistor RL y determine IL



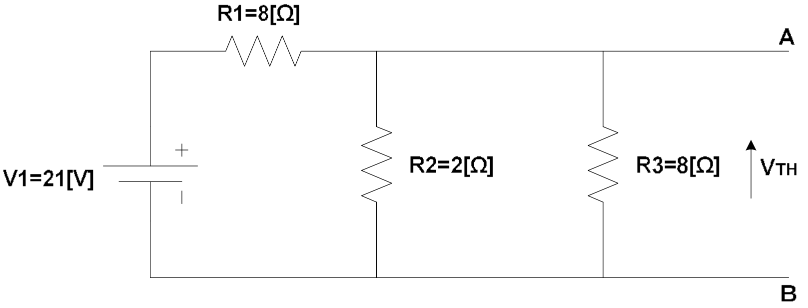
Solución:

1° anular la carga RL y anular la fuente de energía



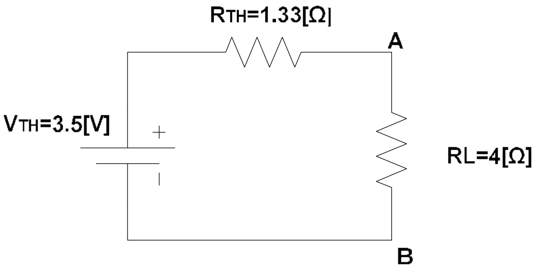
$$R_{TH} = R1 // R2 // R3 = 8//2//8 = \frac{8 \cdot 2}{8+2} // 8 = \frac{16}{10} // 8 = 1.6 // 8 = \frac{1.6 \cdot 8}{1.6+8} = \frac{12.8}{9.6} = 1.33[\Omega]$$

2° buscar voltaje Thevenin



$$V_{TH} = 21 * \frac{8//2}{8//2+8} = 21 \cdot \frac{16/10}{16/10 + 8} = 21 \cdot \frac{1.6}{1.6+8} = \frac{33.6}{9.6} = 3.5[V]$$

3° cálculo de intensidad que circula por RL



$$IL = \frac{3.5}{1.33+4} = \frac{3.5}{5.33} = 0.656A$$